Science Journal of Circuits, Systems and Signal Processing
2017; 6(5): 44-49
http://www.sciencepublishinggroup.com/j/cssp
doi: 10.11648/j.cssp.20170605.11
ISSN: 2326-9065 (Print); ISSN: 2326-9073 (Online)

Research/Technical Note

One Improvement on Zonotope Guaranteed Parameter Estimation

Wang Jian-hong, Liu Fei-fei, Tang Yan-yuan

School of Electronic Engineering and Automation, Jiangxi University of Science and Technology, Ganzhou, China

Email address:
tanyanyuan@jci.edu.cn (Tang Yan-yuan)

To cite this article:
Wang Jian-hong, Liu Fei-fei, Tang Yan-yuan. One Improvement on Zonotope Guaranteed Parameter Estimation. *Science Journal of Circuits, Systems and Signal Processing*. Vol. 6, No. 5, 2017, pp. 44-49. doi: 10.11648/j.cssp.20170605.11

Received: December 16, 2017; **Accepted**: December 27, 2017; **Published**: January 16, 2018

Abstract: This paper studies the guaranteed state estimation in terms of zonotope, and does some improvements for nonlinear discrete time system with a bounded description of noise and parameters. Firstly we extend the Taylor series with respect to two variables so that the mean value extension which is used to compute an interval enclosure can be improved and extended. Secondly based on the improved mean value extension, a generalization of classical method is proposed as it considers uncertainty in the model of system. Thirdly we give one iterative process in one algorithm to obtain a bound of the exact uncertain state set. Finally the simulation example results confirm the identification theoretical results.

Keywords: Nonlinear System, Set Membership Parameter Estimation, Zonotope

1. Introduction

The automatic control field includes three aspects: prediction, filtering and smoothing. One common property exists in these three aspects is to estimate the state of the system. The difference is that different states of the system at different sample instants are estimated by using different input and output observed variables. State estimation is very important in closed loop feedback control and target tracking process, because some information about the linear system or nonlinear system are included in the state estimation. The problem of state estimation is similar to the system identification theory. Their specific process are formulated as follows: given a mathematical model with respect to the real system and collecting some observed input-output measurements, the state of the real system has to be estimated from these measurements by applying some statistic analysis. Then one example is given to illustrate the importance of the state estimation in target tracking process. The goal of target tracking is to estimate some parameters corresponding to the considered target with lots of observed data which are collected by sensors. These unknown parameters include the information of position and velocity. The target tracking can be defined as follows: using some prior probability knowledge and obtaining the state estimation of the target from observed sequences. The state estimation can be used as the state variable in the movement process. Then the state estimation is of great interest for the next designing flight controller.

The methods used for estimating state are divided into two parts: stochastic methods and deterministic methods. The difference between these two methods is whether the prior information about noise is known. Based on some probabilistic assumptions on noise, the stochastic methods (such as Kalman filter, maximum likelihood and Bayes estimate) apply the minimum mean square state estimation error to obtain the state estimation. But the probabilistic assumptions on noise are not realistic and it means these probabilistic assumptions are not realized in reality. So in order to relax the probabilistic assumptions on noise, the deterministic methods are proposed to assume that the noises are unknown but bounded. This unknown but bounded assumption is weaker than the formal probabilistic assumption. Because it needs not any prior distribution of noise. The common used deterministic method is called set membership estimation [1]. Under set membership estimation, the obtained result is not a numerical value but a guaranteed interval on state actually. That guaranteed interval means that the true state estimation can be included in this interval with one guaranteed accuracy which is assessed by probability inequality. Now research on set membership estimation are very widely, because set membership estimation can be not only applied in linear system, but also in nonlinear system [2]. Then different representations are used to describe that guaranteed interval such as polytope, ellipsoid, parallelotope and zonotope. Each representation has its own advantage and disadvantage. In recent years, guaranteed state estimation with zonotopes are proposed in [3] to introduce the interval mathematics in set membership estimation. From interval analysis, the advantage of zonotopes is that one minkowski sum of two zonotopes is also a zonotope. But this fact does not hold for any other representations. Time varying parameter identification with zonotopes is studied under the framework of bound error identification [4]. The trade-off between the complexity of the zonotopes computation and the accuracy of the estimation is handled efficiently by reducing the radius of zonotope [5]. The complexity of the zonotopes computation can be written as a matrix formulation based on linear matrix inequality. Through solving one eigenvalue problem, some values and vertices of zonotope can be found with convex optimization techniques. As guaranteed state estimation needs observed inputs and outputs, then inputs must be chosen appropriately. Input design for guaranteed state estimation and fault diagnosis is proposed in [6], where covariance analysis is deeply derived. State estimation by zonotopes is great importance in model predictive control based on approximated reachable sets [7].

In this paper, we do some improvements on zonotope guaranteed state estimation. Firstly we extend the Taylor series with respect to two variables so that the mean value extension used to compute an interval enclosure can be improved and extended. Secondly based on the improved mean value extension, a generalization of classical method as it considers uncertainty in the model of system is proposed. Thirdly we give one iterative process in one algorithm to obtain a bound of the exact uncertain state set.

2. Problem Formulation

Consider one uncertain nonlinear discrete time system of the following form.

$$\begin{cases} x_{k+1} = f(x_k, w_k) \\ y_k = g(x_k, v_k) \end{cases} \tag{1}$$

where in equation (1) $x_k \in R^n$ is the state of the system and $y_k \in R^p$ is the measured output vector at sample time k. The vector $w_k \in R^{n_w}$ is the time varying process parameters and process perturbation vector. $v_k \in R^{p_v}$ is the measurement noise vector. Here we assume that the uncertainties and the initial state are bounded by known compact sets.

$$w_k \in W, v_k \in V, x_0 \in X_0$$

To be easy to understand the concepts about zonotope, the definitions of zonotope and its other state sets are given as follows.

Definition 1 (Zonotope). Given a vector $p \in R^n$ and a matrix $H \in R^{n\times m}$, the set

$$p \oplus HB^m = \{p + Hz : z \in B^m\} \tag{2}$$

is called a zonotope of order m. Note that $\oplus$ means the minkowski sum operation.

Definition 2 (Consistent state set). Given system (1) and a measured output y_k, the consistent state set at time k is defined as

$$X_{y_k} = \{x \in R^n : y_k \in g(x, v)\}.$$

Definition 3 (Exact uncertain state set). Consider a system given by (1), the exact uncertain state set X_k is equal to the set of states that are consistent with the measured outputs $y_1, y_2 \cdots y_k$ and the initial state set X_0.

$$X_k = f(X_{k-1}, W) \cap X_{y_k}, k \geq 1 \tag{3}$$

3. Some Improvements on Interval Analysis

From interval arithmetic analysis, the natural interval extension is the fundamental theorem which replaces each occurrence of each variable by its corresponding interval variable. The natural interval extension is a particular and efficient way to compute an interval enclosure. Combining the natural interval extension and the Taylor series expansion, the improved mean value theorem can be obtained.

Theorem 1: Consider a function $f : R^n \to R$ with continuous derivatives about x and w, where $x \in R^n$ and $w \in R^{n_w}$. The two known compact sets X and W are given as the following zonotopes.

$$X = p \oplus HB^m, \quad W = c_w \oplus C_w B^{s_w} \tag{4}$$

Then

$$f(X, W) \subseteq f(p, c_w) \oplus MB^{m+s_w}$$

Proof: When applying the Taylor series expansion with respect to two variables, we obtain the following inclusion relation.

$$\begin{aligned} f(X,W) &\subseteq f(p,c_w) \oplus \Box(\nabla f_x(X,W))(X-p) \\ &\quad \oplus \Box(\nabla f_w(X,W))(W-c_w) \\ &= f(p,c_w) \oplus \Box(\nabla f_x(X,W))HB^m \\ &\quad \oplus \Box(\nabla f_w(X,W))C_w B^{s_w} \end{aligned} \tag{5}$$

where $\Box$ means the natural interval extension, as $x_k \in R^n$, then $f(p,c_w)$ is a vector. It means that $f(p,c_w) \in R^n$. Three matrices are also defined as.

$$M_1 = \square(\nabla f_x(X,W))H, M_2 = \square(\nabla f_w(X,W))C_w$$
$$M = [M_1, M_2]$$

Equation (5) can be continued to compute.

$$\begin{aligned} f(X,W) &\subseteq f(p,c_w) \oplus M_1 B^m \oplus M_2 B^{s_w} \\ &= f(p,c_w) \oplus [M_1, M_2] \begin{bmatrix} B^m \\ B^{s_w} \end{bmatrix} \\ &= f(p,c_w) \oplus MB^{m+s_w} \end{aligned} \tag{6}$$

From equation (6), we see that the state set is included in one zonotope. This improved mean value extension uses the continuous derivative in w, because there are two stochastic variables x and w in the state equation.

Consider the following interval extension:

A zonotope $q \oplus SB^d$ such that $f(p,c_w) \subseteq q \oplus SB^d$

Then applying the above interval extension in equation (6), we obtain one zonotope which includes the state set. It means that.

$$\begin{cases} f(X,W) \subseteq q \oplus SB^d \oplus MB^{m+s_w} \\ \quad = q \oplus [S \quad M] \begin{bmatrix} B^d \\ B^{m+s_w} \end{bmatrix} \\ \quad = q \oplus H_q B^{d+m+s_w} \\ H_q = [S \quad M] \end{cases} \tag{7}$$

The zonotopoe $q \oplus SB^d$ can be obtained by means of a natural interval extension of $f(p,c_w)$. Now comparing equation (7) and theorem 4 from [8], the difference is that the Taylor series expansion with two variables is used here. As the stochastic variable w is considered in the derivation process, the obtained zonotope is more appropriate than the one in [9]. But here no doubt that the computational complexity is increased. In order to reduce the computational complexity, the strategy of model reduction can be used to achieve this goal.

In set membership estimation theory, the computation of the exact uncertain state set is very difficult. In practice, the state set is approximated by conservative outer bounds to reduce the computation complexity. This section presents one iterative method to compute an outer approximation using a zonotope. Consider an outer bound of the exact uncertain state set, denoted as $\hat{X}_{k-1}$, is available at time instant $k-1$. Similarly a measured output vector y_k is obtained at time instant k. Then the iterative guaranteed state estimation algorithm can be given as follows.

Iterative guaranteed state estimation algorithm:

Step 1: Given system (1), assume that the initial state x_0 is bounded by a known compact set:

$$x_0 \in X_0 \subseteq q_0 \oplus H_{q_0} B^{d+m+s_w}$$

Compute a state set at time instant k as

$$\bar{X}_k \subseteq q_{k-1} \oplus H_{q_{k-1}} B^{d+m+s_w} \tag{8}$$

where each variable is defined as.

$$\begin{aligned} H_{q_{k-1}} &= [M_{1(k-1)} \quad M_{2(k-1)}] \\ M_{1(k-1)} &= \square(\nabla f_{x_{(k-1)}}(X_{(k-1)}, W_{(k-1)}))H \\ M_2 &= \square(\nabla f_{w_{(k-1)}}(X_{(k-1)}, W_{(k-1)}))C_w \end{aligned} \tag{9}$$

Step 2: For $i = 1, 2 \cdots N$

Compute an outer bound of the consistent state set $X_{y_{k/i}}$ and $\hat{X}_{k/i-1}$ with $\hat{X}_{k/0} = X_0$

Step 3: Compute the intersection operation to obtain the state set $\bar{X}_k \cap X_{y_{k/i}}$.

End of algorithm

The iteration process is started from initial time. The iteration means that the state set at next time instant is dependent of the above state and the above measured output variable. This iteration process is similar to the classical Kalman filter theory. It also is divided into prediction step, measurement step and correction step.

4. Simulation Example

Now in this section we apply our iterative correlation tuning control approach in flight simulation to design one PID controller. Flight simulation is a speed servo system with high precision position. Flight simulation with six-degrees of freedom is seen in Figure 1.

Figure 1. *Flight simulation with six-degrees of freedom.*

The driven element of flight simulation is an electric motor, and the essence of the control structure in flight simulation is a closed loop system corresponding to the position or speed of that electric motor. According to the analysis of the servo control system, one negative feedback part is added to reduce the sensitivity in the closed loop system, while the cascade regulator is introduced in each feedback control structure in order to reduce the dependence on the electric motor's parameter.

Here we give an example about the pitch position tracking loop from flight simulation to verify the feasibility of our iterative correlation tuning control approach in precision servo control system. In the closed loop system of flight simulation, the photoelectric encoder is mounted on the outer pitch frame, the angular position signal collected at outer pitch frame is regarded as the position feedback part. After the difference between two angular positions goes through the position correlation part and power amplifier part, then this difference will make the electric motor start to rotate. The pitch position tracking loop from flight simulation is simplified in Figure 2.

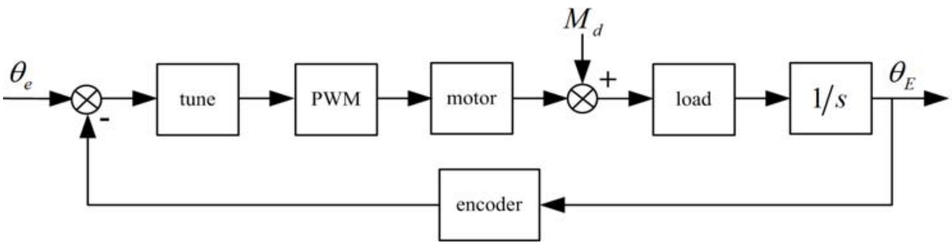

Figure 2. *The simplified pitch position tracking loop.*

In Figure 3 the input signal is the relative angular signal of inner pitch loop, and this input signal is collected by one photoelectric encoder which is located in inner pitch frame. It means that one photoelectric encoder collects the angular position signal to form one position feedback part. The transfer function model of that simplified pitch position tracking loop can be seen in Figure 3.

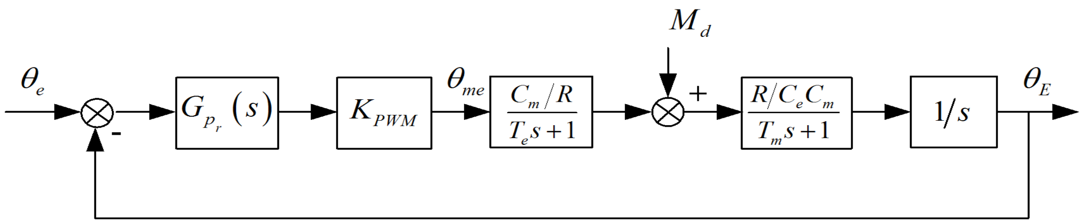

Figure 3. *The transfer function model of that simplified pitch position tracking loop.*

In Figure 4 we regard the encoder as a constant and merge it in the power amplifier, then the close loop system is an unit feedback. θ_{me} is the input signal with respect to the electric motor, the controller in this position tracking loop is the classical PID controller. The linear combination of each proportion (P), integral (I) and differential (D) of that difference is used to control the electric motor. The classical PID control structure is given in Figure 4, $r(t)$ is the chosen input signal, and $y(t)$ is the true output, $e(t)$ is the difference or error.

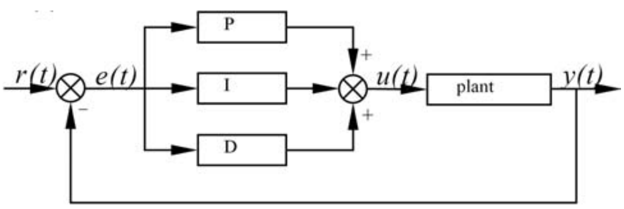

Figure 4. *The classical PID control structure.*

PID controller is a kind of linear controller which includes proportion, integral and differential of the difference or error. Its control law is that.

$$u(t) = K_P\left[e(t) + \frac{1}{T_I}\int_0^t e(\tau)d\tau + \frac{T_D de(t)}{dt}\right]$$

Rewriting the above control law as the transfer function form.

$$G(z) = \frac{U(z)}{E(z)} = K_P(1+\frac{1}{T_I z}+T_D z) = K_P + \frac{K_i}{z} + K_d z$$

where K_P is the proportional coefficient, T_I is the integral coefficient, and T_D is the differential coefficient. After some computation, we obtain the expected transfer function of outer pitch loop.

$$M(z) = \frac{1.725}{(0.0005z+1)(2.41z+1)}, \theta_E = M(z)\theta_e$$

According to the actual parameters of the flight simulation, the electrical and mechanical part of the external pitch frame motor and load are given as follows.

$$\frac{C_m/R}{T_e z+1} = \frac{0.499}{0.1111z+1}, \quad \frac{R/C_e C_m}{T_m z+1} = \frac{2.903}{0.0091z+1}$$

Through comparing this flight simulation example and our iterative correlation tuning control, we regard the motor and load as an integer part, and collect the input-output measured data $\{\theta_{me}, \theta_E\}$ relating this integer part. θ_e is computed by measured data and that expected transfer function $M(s)$.

$$\theta_e = M^{-1}(z)\theta_E$$

Before zonotope parameter identification algorithm is applied to identify those three unknown parameters, let

their true values be that

$$K_p = 7, K_i = 0.5, K_d = 2$$

Applying above six steps to construct a sequence of candidate zonotopes, and after 20 iterations, these candidate zonotopes are given in Figure 5 and Figure 6.

In Figure 5, the black star denotes the optimal two PD parameters as $(K_p, K_d) = (7,2)$., and a sequence of candidate zonotopes generated by zonotope parameter identification algorithm include $(K_p, K_d) = (7,2)$ as their interior point. as these candidate zonotopes have decreasing volumes with iterations, i.e certain contracting properties hold. Generally the two identified parameter estimators corresponding to the PD parameters can be chosen as the center of the smallest zonotope. Further the black star is the optimal ID parameters as $(K_i, K_d) = (0.5,2)$ in Figure 6, and results are similar to them in Figure 5

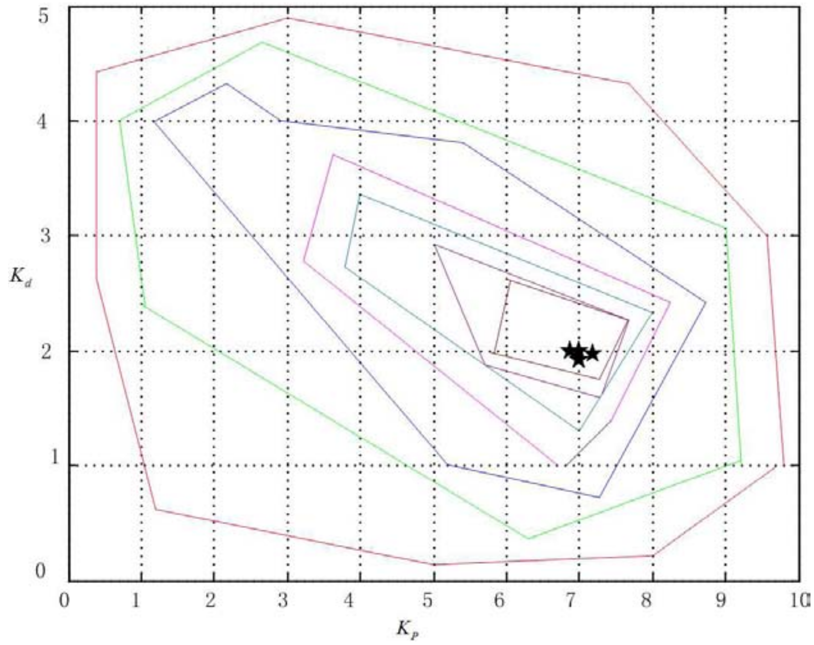

Figure 5. *Candidate zonotopes for PD parameters.*

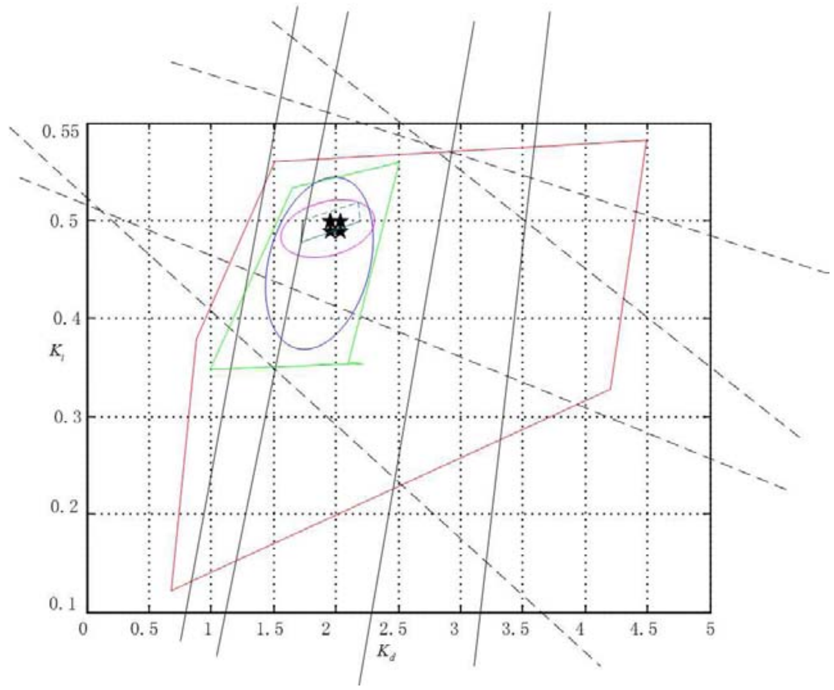

Figure 6. *Candidate zonotopes for ID parameters.*

5. Conclusion

Some improvements on the zonotope guaranteed state estimation method for nonlinear discrete time systems with a bounded description of noise and parameters has been developed. The algorithm gives a state set used to bound the set of all the states that are consistent with the measured output and the given noise. It is different from the classical method, we apply the Taylor series expansion with respect to two variables to get the improved mean value theorem. Furthermore the iteration process is introduced in the guaranteed state estimation algorithm.

Acknowledgements

This work was supported by the Grants from the National Science Foundation of China (no. 61364014).

References

[1] M C Campi, G Calafiore. “Interval predictor models: identification and reliability,” Automatica, vol 45, no. 2, pp. 382-393, 2009.

[2] M C Campi, M Vidyasagar. “Learning with prior information,” IEEE Transaction on Automatic Control, vol 46, no. 11, pp. 1682-1694, 2001.

[3] M Vidyasagar, Rajeeva L Karandikar. “A learning theory approach to system identification and stochastic adaptive control,” Journal of Process Control, vol 18, no. 3, pp. 421-430, 2008.

[4] M C Campi, P R Kumar. “Learning dynamical systems in a stationary environment,” Systems & Control Letters, vol 34, no. 3, pp. 125-132, 1998.

[5] G Calafiore, M C Campi. “Uncertain convex programs: randomized solutions and confidence levels,” Mathematical Programming, vol 102, no. 11, pp. 25-46, 2005.

[6] M Milanese, C Novara. “Set membership identification of nonlinear systems,” Automatica, vol 40, no. 6, pp. 957-975, 2004.

[7] T Alamo, J M Bravo, E F Camacho. “Guaranteed state estimation by zonotopes,” Automatica, vol 41, no. 6, pp. 1035-1043, 2005.

[8] J M Bravo, T Alamo, E F Camacho. “Bounded error identification of systems with time varying parameters,” IEEE Transaction on Automatic Control, vol 51, no. 7, pp. 1144-1150, 2006.

[9] J M Bravo, A Suarez, M Vasallo. “Slide window bounded error tome varying systems identification,” IEEE Transaction on Automatic Control, vol 61, no. 8, pp. 2282-2287, 2016.